The (local) Lifting Problem for Curves

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Local Actions and Lifting

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Definition - lifting local actions

A local action *lifts to char 0* if there exists a finite extension $R/W(k)$ and an action of G on $R[[Z]]$ that reduces to the action of G on $k[[z]]$.

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Question - Local Lifting Problem

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Results

- Local-to-Global principle connects this problem to curves.
- If $(p, |G|) = 1$ then all local actions lift.
- Exist bounds on $|G|$ that obstruct lifting.

Local Lifting Problem

k is algebraically closed of char $p > 0$, $\xi_p \in \mathbb{C}$ a pth root of unity.

Example

- Let $\sigma \in \mathrm{Aut}_k(k[[t]])$ given by $\sigma(t) = \frac{t}{t+1}$. Then σ has order p (think transform with matrix $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$).

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- Can lift action to char 0 using $\tilde{\sigma}(t) = \frac{t}{t+\xi_p}$ (think transform $\begin{pmatrix} 1 & 0 \\ 1 & \xi_p \end{pmatrix}$)).

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- Note: needed to extend to $W(k)[\xi_p]$.

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Thank You!